

# OPTICS

## ALL THE POSSIBLE FORMULAE

- Relation between focal length and radius of curvature of a mirror/lens,  $f = R/2$
- Mirror formula:  $\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$
- Magnification produced by a mirror:  $m = -\frac{v}{u} = -\frac{f}{u-f}$
- Snell's law:  $\frac{\sin i}{\sin r} = {}^1n_2 = \frac{n_2}{n_1}$
- ${}^2n_1 = \frac{1}{{}^1n_2}$
- $n = \frac{c}{v} = \frac{\text{speed of light in vacuum}}{\text{speed of light in a medium}} = \frac{\lambda_{air}}{\lambda_{medium}}$
- If object is in medium of refractive index  $n$ , then  $n = \frac{\text{real depth}}{\text{apparent depth}} = \frac{t}{t_{app}}$
- Apparent shift,  $x = t - \frac{t}{n} = t\left(1 - \frac{1}{n}\right)$
- Critical angle for total internal reflection:  $\sin C = \frac{1}{n} = \frac{1}{n_d}$
- Refraction at spherical (convex) surface: For object in rarer medium and real image in denser medium, the formula is  $\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$  where  $n_2$  &  $n_1$  are the refractive indices of denser and rarer media.
- Lens formula:  $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$
- Linear magnification produced by a lens:  $m = \frac{I}{O} = \frac{v}{u}$
- Lens maker's formula:  $\frac{1}{f} = \frac{1}{v} - \frac{1}{u} = ({}^a n_g - 1) \left[ \frac{1}{R_1} - \frac{1}{R_2} \right] = (n - 1) \left[ \frac{1}{R_1} - \frac{1}{R_2} \right]$
- Power of a lens:  $P = \frac{1}{f}$  diopter (f is in metre)
- Lenses in contact:  $\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$  or  $P = P_1 + P_2$
- Focal length of lens in liquid:  $f_l = \frac{n_g - 1}{n_l - 1} \times f_a$
- Refraction through a prism:  $r_1 + r_2 = A$  and  $i + e = A + \delta$  where A is angle of prism and  $\delta$  is angle of deviation.
- For minimum deviation,  $i = e = i$  and  $r_1 + r_2 = r$ . Therefore,  $\delta_m = 2i - A$
- Refractive Index of the material of prism:  $n = \frac{\sin i}{\sin r} = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}$
- For a thin prism:  $\delta = (n - 1)A$
- Angular dispersion  $= \delta_V - \delta_R$
- Dispersive power,  $\omega = \frac{\delta_V - \delta_R}{\delta_Y} = \frac{n_V - n_R}{n_Y - 1}$
- Simple microscope: Magnifying power  $M = 1 + \frac{D}{f}$  (if final image is at D)  
 $= \frac{D}{f}$  (if final image is at infinity)
- Compound microscope:
  - i) Magnification  $M = m_o m_e$
  - ii) Magnification  $M = -\frac{v_o}{u_o} \left\{ 1 + \frac{D}{f_e} \right\} \approx -\frac{L}{f_o} \left\{ 1 + \frac{D}{f_e} \right\}$  (for final image at D)
  - ii) Magnification  $M = -\frac{v_o}{u_o} \left\{ \frac{D}{f_e} \right\} \approx -\frac{L}{f_o} \left\{ \frac{D}{f_e} \right\}$  (for final image at infinity)
- Astronomical Telescope:
  - i)  $M = -\frac{f_o}{f_e}$  and  $L = f_o + f_e$  (for final image at infinity)

$$\text{ii) } M = -\frac{f_o}{f_e} \left\{ 1 + \frac{f_e}{D} \right\} \text{ and } L = f_o + u_e \text{ (for final image at D)}$$

• Resolving power:

i) **For microscope:** - The resolving power is the reciprocal of limit of resolution or separation between two points such that they are distinct. So, the resolving power is given by  $\text{R.P.} = \frac{1}{d} = \frac{2n \sin\theta}{\lambda}$

Here,  $d = \frac{\lambda}{2n \sin\theta}$  is limit of resolution,  $n \sin\theta$  is numerical aperture and  $\theta$  is the well resolved semi-angle of cone of light rays of wavelength  $\lambda$  entering the microscope.

ii) **For telescope:** - The resolving power is the reciprocal of angular limit of resolution or angle subtended between two points such that they are distinct. So, the resolving power is given by  $\text{R.P.} = \frac{1}{d\theta} = \frac{a}{1.22 \lambda}$

Here,  $d\theta = \frac{1.22 \lambda}{a}$  is the angular limit of resolution, 'a' is the aperture or diameter of objective lens.

- The distance for which ray optics is good approximation for an aperture D and wavelength  $\lambda$  is called Fresnel distance, given by  $Z_F = \frac{D^2}{\lambda}$ .

• Interference of light:-

i) If two waves of same intensity  $I_0$  interfere, then the resultant intensity will be  $I = 4 I_0 \cos^2 \frac{\phi}{2}$  where  $\phi$  is the initial phase difference between the waves.

ii) Resultant intensity at a point in the region of superposition is

$$I = a_1^2 + a_2^2 + 2a_1 a_2 \cos\phi = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos\phi \text{ where } I_1 = a_1^2 \text{ is the intensity of one wave \& } I_2 = a_2^2 \text{ is the intensity of other wave.}$$

iii) Condition for maxima: - Phase difference  $\phi = 2n\pi$  & path difference  $\Delta = n\lambda$  where  $n = 0, 1, 2, 3, \dots$

iv) Condition for minima: - Phase difference  $\phi = (2n-1)\pi$  & Path difference  $\Delta = (2n-1)\frac{\lambda}{2}$  where  $n = 0, 1, 2, 3, \dots$

v) Fringe width  $\beta = \frac{D\lambda}{d}$  where D = distance between the slits & the screen, d = separation between the slits and  $\lambda$  is the wavelength of light used.

vi) Angular fringe width,  $\beta_\theta = \frac{\beta}{D} = \frac{\lambda}{d}$

vii) Minimum amplitude,  $A_{\min} = (a_1 - a_2)$

viii) Minimum intensity,  $I_{\min} = (a_1 - a_2)^2 = I_1 + I_2 - 2\sqrt{I_1 I_2}$

ix) Position of  $n^{\text{th}}$  maxima,  $y_n = \frac{nD\lambda}{d}$

x) Position of  $n^{\text{th}}$  minima,  $y_n = (n - 1/2)\frac{D\lambda}{d}$

• Diffraction of light: -

i) The condition for the position of  $n^{\text{th}}$  minima:  $d \sin\theta = n\lambda$  where d is the width of slit,  $\theta$  is angle of diffraction and  $\lambda$  is the wavelength of light used.

ii) Linear half-width of central maximum:  $y = \frac{D\lambda}{d}$

iii) Total linear width of central maximum:  $\beta_0$  or  $2y = \frac{2D\lambda}{d}$

• Polarisation of light:-

i) Brewster's law:-  $n = \tan i_p$

ii) Malus law:  $I = I_0 \cos^2\theta$

### SHORTCUT FORMULAE

1. Magnification in lens:  $m = \frac{f}{f+u} = \frac{f-v}{u}$
2. Magnification in combination of lenses:  $m = m_1 m_2$
3. If two thin lenses have distance (x) between them, then,  $\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{x}{f_1 f_2}$
4. For a fixed distance D between object and image for its real image,  $f = \frac{D^2 - x^2}{4D}$  where x is the separation between two positions of lens. For maximum f,  $x = 0$ ;  $\therefore f_{\max} = D/4$
5. Focal length of lens in liquid:  $f_l = \frac{n_g - 1}{n_l - 1} \times f_a$
6. Relation between refractive index (n) and wavelength of light (Cauchy's formula)  

$$n = a + \frac{b}{\lambda^2} + \frac{c}{\lambda^4} + \dots$$
7. For thin prism:  $\delta = (n - 1)A$
8. Rayleigh's scattering criteria: Intensity of scattered light,  $I_s \propto \frac{1}{\lambda^4}$
9. In interference, the ratio of maximum intensity to minimum intensity,  $\frac{I_{\max}}{I_{\min}} = \frac{(a_1 + a_2)^2}{(a_1 - a_2)^2}$
10. In interference, the relation between slit width (w) , intensity (I) and amplitude (a):  

$$\frac{w_1}{w_2} = \frac{I_1}{I_2} = \frac{(a_1)^2}{(a_2)^2}$$
11. The angular width of each fringe in interference pattern,  $\Delta\theta = \frac{\beta}{D} = \frac{\lambda}{d}$
12. In diffraction, Fresnel distance  $Z_F = \frac{d^2}{\lambda}$  where 'd' is slit width.
13. In reflecting type telescope: Magnifying power,  $M = \frac{f_o}{f_e}$  and Brightness,  $B = \frac{D^2}{d^2}$  where D is the diameter of the objective lens and d is the diameter of the pupil of eye.
14. In interference, Fringe width  $\beta \propto \lambda$  ,  $\beta \propto D$  and  $\beta \propto 1/\text{separation between the slits (d)}$

### NUMERICAL PROBLEMS (UNIT – OPTICS)

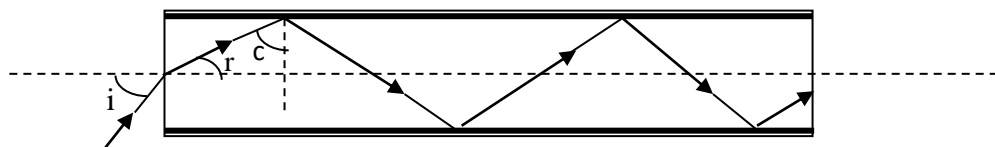
#### LEVEL – I

1. An object is placed at the principal focus of a concave lens of focal length f. Where will its image be formed? (1)
2. A prism of angle  $60^\circ$  gives a minimum deviation of  $30^\circ$ . What is the refractive index of the material of the prism? (1)
3. An equi-convex lens has refractive index 1.5. Write its focal length in terms of radius of curvature R. (1)
4. Estimate the distance for which ray optics is good approximation for an aperture of 4mm and wavelength 400nm. (1)
5. What is Brewster's angle for air to glass transition? Refractive index of glass = 1.5. (1)
6. In Young's double slit experiment the slits are separated by 0.28mm and the screen is placed 1.4m away. The distance of 4<sup>th</sup> bright fringe is measured to be 1.2cm. Determine the wavelength of light used in this experiment. (2)

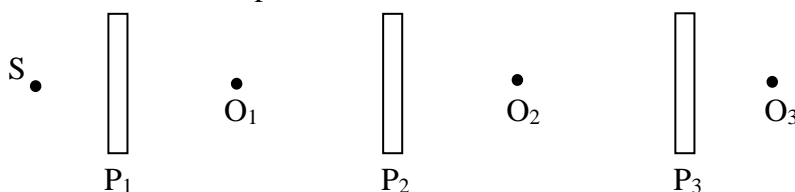
7. An astronomical telescope uses two lenses of powers 10D and 1D. What is its magnifying power in normal adjustment? (2)
8. Light of wavelength 500nm falls, from a distant source, on a slit 0.5mm wide. Find the distance between the two dark bands, on either side of the central bright band of the diffraction pattern observed, on a screen placed 2m from the slits. (2)
9. An illuminated object and a screen are placed 90cm apart. Determine the focal length and nature of the lens required to produce a clear image on the screen, twice the size of the object. (2)
10. The near vision of an average person is 25cm. To view an object with an angular magnification of 10, what should be the power of the microscope? (2)

**LEVEL – II**

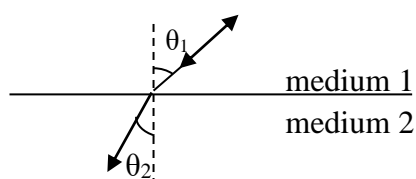
1. A mirror is turned through  $15^\circ$ . Through what angle will the reflected ray turn? (1)
2. Velocity of light in a liquid is  $1.5 \times 10^8$  m/s and in air, it is  $3 \times 10^8$  m/s. If a ray of light passes from liquid into the air, calculate the value of critical angle. (1)
3. Why does a convex lens of glass of refractive index 1.5 behave as a diverging lens when immersed in carbon disulphide of refractive index 1.65? (1)
4. Find the angular dispersion produced by a thin prism of  $5^\circ$  having refractive index for red light 1.5 and for violet light 1.6. (1)
5. If a person uses spectacles of power +1.0D, what is the nearest distance of distinct vision for him? Given that near point of the person is 75cm from the eye. (2)
6. In Young's double slit experiment, light waves of wavelength  $5.4 \times 10^{-7}$  m and  $6.85 \times 10^{-8}$  m are used in turn keeping the same geometry. Compare the fringe width in the two cases. (2)
7. If the two slits in Young's experiment have width ratio 1:4, deduce the ratio of intensity at maxima and minima in the interference pattern. (2)
8. Figure shows a cross-section of a 'light pipe' made of a glass fibre of refractive index 1.68. The outer covering of the pipe is made of a material of refractive index 1.44. What is the range of the angles of incident rays with the axis of the pipe for which total reflections inside the pipe take place as shown. (3)



9. Three identical Polaroid sheets  $P_1$ ,  $P_2$  and  $P_3$  are oriented so that the (pass) axis of  $P_2$  and  $P_3$  are at angles of  $60^\circ$  and  $90^\circ$  respectively, with respect to the pass axis of  $P_1$ . A monochromatic source,  $S$ , of intensity  $I_0$ , is kept in front of the Polaroid sheet  $P_1$ . Find the intensity of this light, as observed by observers  $O_1$ ,  $O_2$  and  $O_3$ , positioned as shown below. (3)



10. Light of wavelength  $\lambda_1$  propagates from medium 1 incident at angle  $\theta_1$ . The angle inside medium 2 is  $\theta_2$ . What is its wavelength in medium 2? (3)



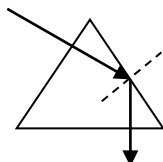
### LEVEL – III

1. A ray of light is incident on a concave mirror after passing through its center of curvature. What is the value of angle of reflection? (1)
2. What is the ratio of fringe width for dark and bright fringes in Young's double slit experiment? (1)
3. A dentist uses a small concave mirror of focal length 16mm to view a cavity in the tooth of a patient by holding the mirror at a distance of 8mm from the cavity. Calculate the magnification. (2)
4. Show that for a concave mirror, a virtual object forms a real image which is always diminished. (2)
5. A point source of light is placed at the bottom of a lake with refractive index  $4/3$ . Show that only 17% light can emerge out of the water surface. (2)
6. Why does violet colour deviate more than red in prism? (2)
7. A ray of light is incident at an angle of incidence 'i' on one surface of a prism of small angle 'A' and it is found to emerge normally from the opposite surface. If the refractive index of the material of the prism is 'n', calculate the angle of incidence. (3)
8. Calculate the number of fringes displaced when a thin sheet of refractive index 'n' and thickness 't' is introduced in the path of one of the interfering rays. (3)
9. A few coloured fringes, around a central white region, are observed on the screen when the source of monochromatic light is replaced by white light in Young's double slit experiment. Give reason. (3)
10. Light from two sources has intensity ratio 1:9 and is monochromatic. The light is made to superpose. What will be the resultant intensity obtained if the sources are (i) incoherent & (ii) coherent? (3)

**ANSWERS / SOLUTIONS / KEY**

**KEY for Worksheet-1 (Ray Optics)**

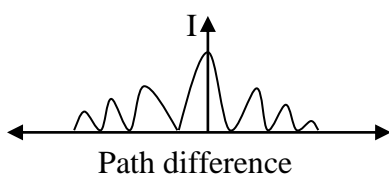
1. Power of lens increases.
2. Correct definition.
3. For microscope 1cm and 3cm; for telescope 100cm and 1cm.
4. At infinity.
- 5.



6. Nature of the prism material and choice of extreme colours for which dispersive power is to be measured.
7. The image formed by plane mirror should be at 24cm behind the mirror or 12cm behind the convex mirror. For no parallax between the images formed by the two mirrors, the image formed by the convex mirror should also be at I. Therefore, for convex mirror  
 $u = -36\text{cm}$ ,  $v = 12\text{cm}$ . So,  $f = 18\text{ cm}$  and  $R = 36\text{cm}$
8. Neat and labeled diagram.
9. Correct ray diagram and derivation of the equation  $M = - f_o / f_e$
10. The remedial lens should make the object at infinity appear at the far point. Therefore, for objects at infinity,  $u = -\infty$ .  
 Far point distance of the defective eye,  $v = -80\text{cm}$   
 By thin lens formula,  $f = -0.80\text{m}$ . So  $P = -1.25\text{D}$
11. Correct ray diagram and derivation.
12. Correct ray diagram and derivation.

**KEY for Worksheet-2 (Wave Optics)**

1. Sources which emit continuously light waves of the same frequency or wavelength with a zero or constant phase difference between them are called coherent sources.
2. 9:1
3.  $(7/2) \beta$
4. It is the reciprocal of the smallest angular separation between two point objects whose images can be just resolved by it.
5. Path difference,  $d \sin\theta = \lambda$  where 'd' is slit width.
6. From Malus law,  $I = I_0 \cos^2 60^\circ = I_0 / 4$ . Therefore,  $I / I_0 = 1/4$
- 7.



8. One difference and diagrams.
9. Verification of the law of reflection, with a neat and labeled diagram.

10. i) For first minimum of the diffraction pattern,  $d \sin \theta = \lambda$   
 Therefore,  $d = \lambda / \sin \theta = 6500 \times 10^{-10} / \sin 30^\circ = 1.3 \times 10^{-6} \text{ m}$   
 ii) For first secondary maximum of the diffraction pattern,  $d \sin \theta = 3\lambda/2$   
 Therefore,  $d = 3\lambda/2 \sin \theta = 3 \times 6500 \times 10^{-10} / 2 \sin 30^\circ = 1.95 \times 10^{-6} \text{ m}$
11. Definition:- The angle of incidence at which a beam of unpolarised light falling on a transparent surface is reflected as a beam of completely plane polarized light is called polarizing angle.  
 Derivation of the relation,  $n = \tan i_p$  with a neat and labeled diagram.
12. Derivation of the relation,  $\beta = D\lambda/d$  with a neat and labeled diagram.

### KEY/ANSWERS TO NUMERICAL PROBLEMS (UNIT – OPTICS)

#### Level – I

1. Given:  $u = -f$ , and for a concave lens  $f = -f$ ,  $v = ?$   
 Calculations: From lens formula,  $\frac{1}{v} = \frac{1}{f} + \frac{1}{u}$   
 On substituting the values and on simplifying, we get,  $v = -f/2$   
 That is image will be formed between optical centre and focus of lens: towards the side of the object.
2. Given:  $A=60^\circ$ ,  $\delta_m=30^\circ$ ,  $n = ?$   
 Calculations: Use the formula,  $n = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}$  to get  $n = 1.41$
3. Using the formula,  $\frac{1}{f} = (n - 1) \left[ \frac{1}{R_1} - \frac{1}{R_2} \right]$  we get,  $\frac{1}{f} = (1.5 - 1) \left[ \frac{1}{R} - \frac{1}{-R} \right] = \frac{1}{R}$   
 $\therefore f = R$
4. The distance for which ray optics is good approximation for an aperture  $D$  and wavelength  $\lambda$  is called Fresnel distance, given by  $Z_F = \frac{D^2}{\lambda}$ .  
 Given:  $D = 4\text{mm} = 4 \times 10^{-3} \text{ m}$ ,  $\lambda = 400\text{nm} = 400 \times 10^{-10} \text{ m}$ ,  $Z_F = ?$   
 Calculations: Using the above formula, on substituting the values and simplification, we get,  
 $Z_F = 40\text{m}$
5. Given:  $n = 1.5$ ,  $i_p = ?$   
 Calculations: Using the formula,  $n = \tan i_p$ , we get,  $i_p = \tan^{-1}(n) = \tan^{-1}(1.5) = 56.3^\circ$
6. Given:  $d = 0.28\text{mm} = 0.28 \times 10^{-3} \text{ m}$ ,  $D = 1.4\text{m}$ ,  $n = 4$ ,  $y_4 = 1.2\text{cm} = 1.2 \times 10^{-2} \text{ m}$ ,  $\lambda = ?$   
 Calculations: Using the formula for the position of  $n$ th bright fringe,  $y_n = nD\lambda/d$  we get,  
 $\lambda = y_4 d / 4 D$   
 On substituting the values and on simplification we get,  $\lambda = 6 \times 10^{-7} \text{ m} = 600 \text{ nm}$
7.  $M = - \frac{f_o}{f_e} = - \frac{P_e}{P_o} = \frac{-10}{1} = -10$
8. The distance between two dark bands on either side of central bright bands is equal to the total width of bright band and is given by  $\beta_o = \frac{2 D \lambda}{d}$   
 Given:  $D=2\text{m}$ ,  $\lambda=500\text{nm} = 500 \times 10^{-10} \text{ m}$ ,  $d = 0.5\text{mm} = 0.5 \times 10^{-3} \text{ m}$ ,  $\beta_o = ?$   
 Calculations: On substituting the values and on simplification we get,  $\beta_o = 4 \times 10^{-3} \text{ m} = 4 \text{ mm}$
9. Given:  $u + v = 90\text{cm}$  ..... (i)  
 $m = |v| / |u| = 2$  or  $|v| = 2 |u|$  ..... (ii)  
 From (i) and (ii),  $|u| = 30\text{cm}$ ,  $|v| = 60\text{cm}$   
 By sign convention,  $u = -30\text{cm}$ ,  $v = 60\text{cm}$

Substituting the values in equation  $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$  and after simplification we get,  
 $f = 20\text{cm}$  (convex lens)

10. Given:  $D = 25\text{cm}$ ,  $M=10$ ,  $P=?$

Calculations: Using the formula,  $M = D / f$ , we get,  $f = D/M = 25/10 \text{ cm} = 0.025\text{m}$

Now,  $P = 1 / f$  (in m)  $= 1 / 0.025\text{m} = 40\text{D}$

### Level – II

1.  $30^\circ$ , as the reflected ray turns through twice the angle through which mirror is turned.

2.  $n=c/v = 1/\sin C$ , therefore,  $\sin C = v/c = 1.5 \times 10^8 / 3 \times 10^8 = 0.5$

Now,  $C = \sin^{-1}(0.5) = 30^\circ$

3. This is because  $n = \frac{n_g}{n_c} = \frac{1.5}{1.65} < 1$

From lens maker's formula, 'f' becomes negative. Therefore, the lens behaves as a diverging lens.

4. Given:  $A=5^\circ$ ,  $n_r = 1.5$ ,  $n_v = 1.6$ , angular dispersion = ?

Calculations: On substituting the values in equation, angular dispersion  $= (n_v - n_r)A$ , and on simplification, we get, angular dispersion  $= 0.5^\circ = 30'$

5. Given:  $P = 1 \text{ D}$ ,  $f = 100/P = 100/1 = 100\text{cm}$ , nearest distance of distinct vision  $u = ?$ ;  $v = -75\text{cm}$

Calculations: Using lens formula, we get,  $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$$\therefore \frac{1}{u} = \frac{1}{v} - \frac{1}{f} = -\frac{1}{75} - \frac{1}{100} = \frac{-4-3}{300} = -7/300 \quad \text{or} \quad u = -42.9\text{cm}$$

6. Given:  $\lambda_1 = 5.4 \times 10^{-7} \text{ m}$ ,  $\lambda_2 = 6.85 \times 10^{-8} \text{ m}$ ,  $\beta_1/\beta_2 = ?$

Calculations: As  $\beta = \frac{D\lambda}{d}$  and geometry is same i.e.,  $D$  and  $d$  remain same, therefore,

$$\beta_1/\beta_2 = \lambda_1/\lambda_2 = 5.4 \times 10^{-7}/6.85 \times 10^{-8} = 8 \text{ (approximately)}$$

7. Intensity  $\propto$  width ( $w$ ) of slit

Also, intensity  $\propto$  square of the amplitude,  $\therefore \frac{w_1}{w_2} = \frac{I_1}{I_2} = \frac{a^2}{b^2} = \frac{1}{4}$  or  $\frac{a}{b} = \sqrt{\frac{1}{4}} = \frac{1}{2}$  or  $b = 2a$

$$\text{Now, } \frac{I_{max}}{I_{min}} = \frac{(a+b)^2}{(a-b)^2} = \frac{(a+2a)^2}{(a-2a)^2} = \frac{9}{1}$$

8. Given:  $n_2=1.68$ ,  $n_1=1.44$ ,  $i_{max} = ?$

Calculations: As  $n = \frac{n_2}{n_1} = \frac{1}{\sin C} \therefore \sin C = \frac{n_1}{n_2} = \frac{1.44}{1.68} = 0.8571$  So,  $C = \sin^{-1}(0.8571) = 59^\circ$

Total internal reflection would take place when  $i > C$  i.e.,  $i > 59^\circ$  or when  $r < r_{max}$ , where

$$r_{max} = 90^\circ - C = 90^\circ - 59^\circ = 31^\circ$$

As  $\frac{\sin(i)_{max}}{\sin(r)_{max}} = 1.68 \therefore \sin(i)_{max} = 1.68 \sin(r)_{max} = 1.68 \times \sin 31^\circ = 1.68 \times 0.5156 = 0.8662$

$$\therefore i_{max} = \sin^{-1}(0.8662) = 60^\circ$$

9. Intensity observed by  $O_1 = I_0 / 2$

Intensity observed by  $O_2 = (I_0 / 2) \cos^2(60^\circ) = (I_0 / 2) \times (1/2)^2 = I_0 / 8$

Intensity observed by  $O_3 = (I_0 / 8) \cos^2(90^\circ - 60^\circ) = (I_0 / 8) \cos^2(30^\circ) = (I_0 / 8) (\sqrt{3}/2)^2 = (3/32) I_0$

10. Snell's law says,  $v_2 \sin \theta_1 = v_1 \sin \theta_2$

The ratio of wavelengths is equal to the ratio of the speeds of light.

$$\therefore \lambda_1/\lambda_2 = v_1/v_2$$

Or, wavelength in medium 2,  $\lambda_2 = (\sin \theta_2 / \sin \theta_1) \lambda_1$



### LEVEL – III

1.  $0^\circ$ . A ray of through center of curvature is incident normal to the surface of the mirror.
2. 1:1, since the widths of bright and dark bands are equal.
3. Given:  $u = -8\text{mm}$ ,  $f = -16\text{mm}$ ,  $m = ?$

$$\text{Calculations: } \frac{1}{v} = \frac{1}{f} - \frac{1}{u} = \frac{1}{-16} - \frac{1}{-8} = \frac{1}{16} \quad \therefore v = 16 \text{ mm}$$

$$\text{Now, } |m| = v/u = 16/8 = 2$$

4.  $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$ . Here, 'u' is positive, 'f' is negative,  $v = ?$

$$\text{Giving signs to } u, v \text{ and } f, \text{ we have } \frac{1}{v} + \frac{1}{u} = \frac{1}{-f} \quad \text{or} \quad v = -\left\{\frac{fu}{f+u}\right\} \text{ which is negative.}$$

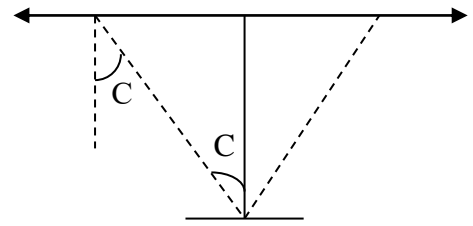
$$\text{Also, } |m| = |v/u| = \frac{1}{\left|\frac{u}{f}+1\right|} < 1 \quad \text{Hence, the image is diminished.}$$

5. The fraction of light energy that can escape is the fraction of the solid angle which allows it to pass without total internal reflection.

Let the critical angle be  $C$ , so that,  $\sin C = \frac{1}{n}$  where 'n' is the refractive index of water.

$$\text{Fraction of solid angle} = \frac{2\pi}{4\pi} (1 - \cos C) = \frac{1}{2} - \frac{1}{2} \sqrt{1 - \sin^2 C}$$

$$= \frac{1}{2} - \frac{1}{2n} \sqrt{n^2 - 1} = \frac{1}{2} - \frac{1 \times 3}{2 \times 4} \sqrt{\left(\frac{4}{3}\right)^2 - 1} = 0.17 = 17\%$$



6. For a prism,  $\delta = (n - 1)A$  and  $n = a + \frac{b}{\lambda^2} + \frac{c}{\lambda^4} + \dots$

As  $\lambda_v < \lambda_R \therefore n_v > n_R$  Hence,  $\delta_v > \delta_R$  So, violet deviates more than red.

7. For refraction through a prism: we have

$$\angle i + \angle e = \angle A + \angle \delta$$

$$\text{But, } \angle e = 0$$

$$\therefore \angle i = \angle A + \angle \delta$$

$$\text{Also, } \delta = (n - 1)A$$

$$\therefore \angle i = A + (n - 1)A = nA$$

8. Let the central maximum shift from  $P_o$  to  $P'_o$ .

The sheet introduces an optical path  $nt$  and decreases air path by 't'.

$$\therefore \text{path difference} = S_2 P'_o - S_1 P'_o = (n - 1)t \quad \dots (i)$$

$$\text{From the figure, } S_2 P'_o - S_1 P'_o = \frac{2xd}{D+D} = \frac{xd}{D} \quad \dots (ii)$$

$$\text{From (i) and (ii), } \frac{xd}{D} = (n - 1)t$$

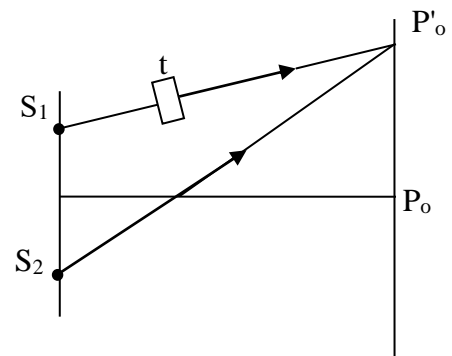
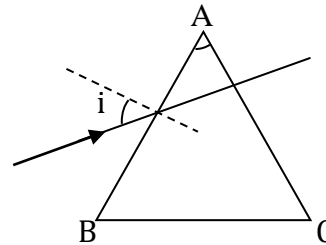
$$\therefore \text{number of fringes shifted, } N = \frac{x}{\beta} = \frac{(n - 1)t}{\lambda}$$

9. For the central maxima, path difference,  $p = n\lambda = 0$  (since  $n = 0$ ) and is independent of  $\lambda$ . Hence, all the colours superpose constructively producing central white fringe.

$$\text{For position of other maxima, } x_n = \frac{nD\lambda}{d}$$

So, the position depends on  $\lambda$ .

As  $\lambda_{\text{red}} > \lambda_{\text{blue}}$ , So, the fringes closest to the central white are blue on either side and farthest are red. After a few fringes, no clear fringe pattern is observed.



10. As  $\frac{I_1}{I_2} = \frac{1}{9}$  (intensity ratio)

Then,  $\frac{a}{b} = \sqrt{\frac{I_1}{I_2}} = \frac{1}{3}$  (amplitude ratio)

If the sources are incoherent, the intensities add up.

i.e., the resultant intensity will be 10. ( $=I_1 + I_2 = a^2 + b^2$ )

If the sources are coherent, we get interference maxima and minima.

At minima, amplitude,  $a_{\min} = 3 - 1 = 2$

$$I_{\min} = (2)^2 = 4$$

At maxima, amplitude,  $a_{\max} = 3 + 1 = 4$

$$I_{\max} = (4)^2 = 16$$

So, the intensity will vary from 4 to 16.

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