Magnetic Effect of Current and Magnetism

Important Formulae

- Biot-Savart law (Magnetic field due to current element)

$$dB = \frac{\mu_0}{4\pi}\frac{I\,dl\sin\theta}{r^2}$$

  - Force acting on a charge moving in a magnetic field

$$F = qvB\sin\theta \qquad \text{or} \qquad \vec{F} = q(\vec{v}\times\vec{B})$$

  - Magnetic field on the axis of a circular current loop

$$B = \frac{\mu_0 N I a^2}{2(r^2 + a^2)^{3/2}}.$$

  - Magnetic field due to an infinitely long straight current carrying wire

$$B = \frac{\mu_0 I}{2\pi r}$$

  - Magnetic field at a point on the axis of a solenoid

$$B = \mu_0\, nI$$

  - Motion of a charged particle in a uniform magnetic field

$$r = \frac{mv}{qB} \qquad T = \frac{2\pi m}{qB}$$

  - Torque on a rectangular coil in a uniform magnetic field

$$\tau = NIBA\sin\theta$$

  - Force per unit length acting on each of the two straight parallel metallic conductors carrying current

$$f = \frac{F_2}{l} = \frac{\mu_0 I_1 I_2}{2\pi r}$$

  - Deflection in moving coil galvanometer

$$\alpha = \frac{NBAI}{k}$$

  - Conversion of galvanometer into ammeter and voltmeter

$$S = \frac{I_g G}{(I - I_g)} \qquad R = \frac{V}{I_g} - R_G$$

  - Elements of Earth's magnetic field

$$H = B_E\cos\theta \qquad V = B_E\sin\theta$$

  - P.E. of a magnetic dipole in a uniform magnetic field

$$U = -mB\cos\theta$$

  - Magnetic dipole moment of a revolving electron

$$m = \frac{evr}{2} = n\left(\frac{eh}{4\pi m_e}\right)$$

- Magnetising field intensity

$$H = nI$$

- Intensity of magnetization

$$M = \frac{m}{V}$$

Direct Formula Based Numerical (Level-I)

Q. 1. The vertical component of Earth's magnetic field at a place is $\sqrt{3}$ times the horizontal component. What is the value of angle of dip at this place?

Q. 2. A short bar magnet placed with its axis at 30^o to a uniform magnetic field of 0.2 T experiences a torque of 0.06 Nm. (i) Calculate the magnetic moment of the magnet. (ii) Find out what orientation of the magnet corresponds to its stable equilibrium in the magnetic field.

Q. 3. A solenoid has a core of a material with relative permeability 400. The winding of the solenoid are insulated from the core and carry a current of 2 A. If the number of turns is 1000 per meter, calculate (i) H, (ii) M and (iii) B.

Q. 4. In the Bohr model of hydrogen atom, an electron revolves around the nucleus in a circular orbit of radius 5.11×10^{-11} m at a frequency of 6.8×10^{15} Hz. What is the magnetic field at the centre of the orbit?

Q. 5. Two long parallel wires carrying currents 8 A and 5 A in the same direction are separated by a distance of 4 cm. Estimate the force on 10 cm length of one wire due to the other wire.

Q. 6. A solenoid of 500 turns per meter is carrying a current of 3 *A*. Its core is made of iron of relative permeability 5000. Determine the magnitudes of magnetic intensity and magnetic field inside the core.

Q.7. A long straight wire carries a current of 35 A. What is the magnitude of magnetic field at a point 20 cm from the wire?

Q. 8. A circular coil of wire consisting of 100 turns, each of radius 8 cm carries a current of 0.4 A. What is the magnitude of magnetic field at its centre?

Q. 9. A closely wound solenoid 80 cm long has 5 layers of winding of 400 turns each. If the current carried is 8 A, estimate the magnetic field inside the solenoid near its centre.

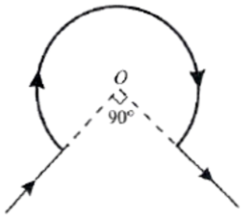
Q. 10. A galvanometer of coil resistance 50 Ω shows full scale deflection for a current of 5 mA. How can it be converted into a voltmeter of range 0 to 15 V?

Moderate Difficulty Level Numerical (Level-II)

Q. 1. Two identical magnetic dipoles of magnetic moment $1Am^2$ each are placed at a separation of 2 m with their axes perpendicular to each other. What is the resultant magnetic field at a point mid-way between the dipoles?

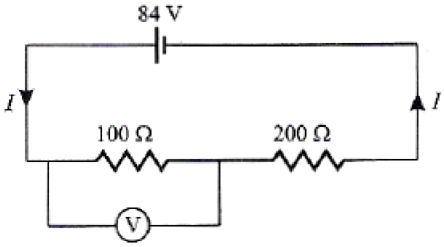
Q. 2. A magnetic dipole is under the influence of two magnetic fields. The angle between the field direction is 60^o and one of the field has a magnitude of 1.2×10^{-2} T. If the dipole comes to stable equilibrium at an angle of 15^o with this field, what is the magnitude of the other field?

Q. 3. The wire shown below carries a current *I*. Determine magnetic field at the centre. Radius of circular section is *R*.



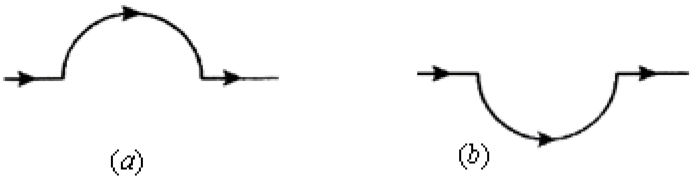
Q. 4. To increase the current sensitivity of a moving coil galvanometer by 50%, its resistance is increased so that the new resistance becomes twice its initial resistance. By what factor does its voltage sensitivity change?

Q. 5. A voltmeter *V* of resistance 400 Ω is used to measure the potential difference across a 100 Ω resistor as shown. What will be the reading of the voltmeter? Also find the potential difference across the resistor before the voltmeter is connected.



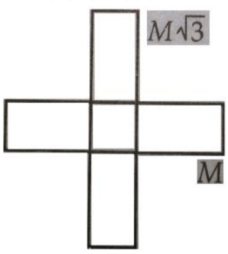
Q.6. A compass needle of magnetic moment 60 Am^2 pointing geographical north at a place where the horizontal component of earth's magnetic field is 40 $\mu Wb/m^2$, experiences a torque of 1.2×10^{-3} Nm. What is the declination of the place?

Q. 7. A straight wire carrying a currant of 12 *A* is bent into a semicircular arc of radius 2 *cm* as shown below. What is the magnitude and direction of magnetic field at the centre of the arc? Would the answer change if it bent in the opposite way as shown in another figure?



Q.8. A straight wire of mass 200 g and length 1.5 m carries a current of 2 A. It is suspended in mid air by a uniform horizontal magnetic field. What is the magnitude of the field?

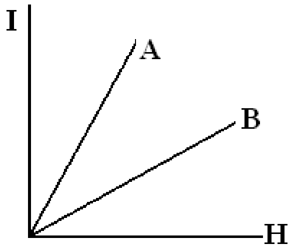
Q. 9. Two magnets of magnetic moments *M* and $M\sqrt{3}$ are joined to form a cross. The combination is suspended in a uniform magnetic field *B*. The magnetic moment *M* now makes an angle θ with the field direction. Find the value of θ.



Q. 10. The following figure shows the variation of intensity of magnetization versus the applied magnetic field intensity for two magnetic materials *A* and *B*.

(i) Identify the materials.

(ii) For the material *B*, plot the variation of intensity of magnetization versus temperature.

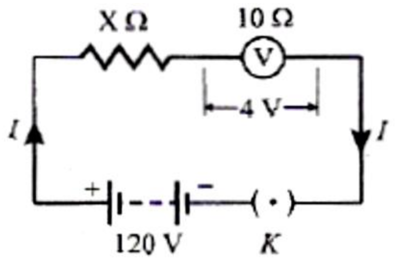


Challenging Level Numerical(Level-II)

Q. 1. A wire carrying a steady current is first bent in form of a circular coil of one turn and then in form of a circular coil of two turns. Fine the ratio of magnetic fields at the centers of the two coils.

Q. 2. A galvanometer gives deflection of 10 division per mA. The resistance of galvanometer is 60 Ω. If a shunt of 2.5 Ω is connected to the galvanometer and there are 50 divisions on the galvanometer scale, what maximum current can this galvanometer read?

Q. 3. A source of 120 *V* is connected to a large resistance *X*. A voltmeter of resistance 10 kΩ placed in series reads 4 *V*. What is the value of *X*? Why is voltmeter used instead of an ammeter?



Q.4. Show that the magnetic field on the axis of a current carrying circular coil of radius *r* at a distance *x* from the centre of the coil is smaller by the fraction $3x^2/2r^2$ than the field at the centre of the coil.

Q. 5. A galvanometer can be converted into a voltmeter to measure upto

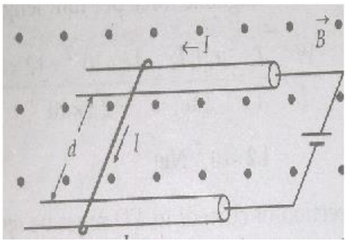
(i) V volt by connecting a resistance R_1 in series with coil.

(ii) V/2 volt by connecting a resistance R_2 in series with its coil.

Find the resistance in terms of R_1 and R_2 required to convert it into a voltmeter that can read upto '2V' volt.

Q. 6. A coil in the shape of an equilateral triangle of side 0.02 m is suspended from a vertex such that it is hanging in a vertical plane between the pole pieces of permanent magnet producing a horizontal magnetic field 5×10^{-2} T. Find the couple acting on the coil when a current of 0.1 A is passed through it and the magnetic field is parallel to its plane.

Q. 7. A metal wire of mass *m* slides without friction on two horizontal rails spaced at distance *d* apart as shown. The rails are situated in a uniform magnetic field *B*, directed vertically upwards and a battery is sending a current *I* through them. Find the velocity of the wire as a function of time assuming it to be at rest initially.



Q. 8. A hollow cylindrical conductor of radii *a* and *b* carries a current *I* uniformly spread over its cross section. Find the magnetic field *B* for points inside the body of the conductor at a distance *r* from the axis.

Q. 9. A straight horizontal conducting rod of length 0.6 m and mass 60 g is suspended by two vertical wires at its ends. A current of 5 A is set up in the rod through its wires. What magnetic field should be set up normal to the conductor in order that the tension in the wires is zero? What will be total tension in the wires if the direction of current is reversed?

Q. 10. A beam of proton passes undeflected through a region of mutually perpendicular electric and magnetic fields of magnitudes 50 kVm^{-1} and 100 mT respectively. Calculate the velocity of the beam. If this beam of current *I* strikes a screen with *n* protons per second, Find the force on the screen.

Level-I

Ans1. $\tan\delta = \frac{B_V}{B_H} = \frac{\sqrt{3}B_H}{B_H} = \sqrt{3}$

$\delta = 60^0$

Ans2. (i) $M = \frac{\tau}{B\sin\theta} = \frac{0.06}{0.2\sin 60^0} = \frac{0.06}{0.2\times0.5} = 0.6\,Am^2$

(ii) The P.E. of a magnetic dipole in a uniform magnetic field is

$U = -mB\cos\theta$

In stable equilibrium the P.E. is minimum, so $\cos\theta = 1 \quad or \quad \theta = 0^0$

Hence the bar magnet will be in stable equilibrium when its magnetic moment is parallel to the magnetic field.

Ans3. (i) $H = nI = 1000\times2 = 2000\,Am^{-1}$

(ii) $B = \mu H = \mu_0\mu_r H = 4\pi\times10^{-7}\times400\times2000 = 1T$

(iii) $M = \chi_m H = (\mu_r - 1)H = (400-1)\times2000 = 8\times10^5\,Am^{-1}$

Ans4. $B = \frac{\mu_0 I}{2r} = \frac{\mu_0 f e}{2r} = \frac{4\pi\times10^{-7}\times6.8\times10^{15}\times1.6\times10^{-19}}{2\times5.11\times10^{-11}} = 13.4T$

Ans5. $F = \frac{\mu_0 I_1 I_2}{2\pi r}l = \frac{4\pi\times10^{-7}\times8\times5\times0.1}{2\pi\times0.04} = 2\times10^{-5}\,N$

Ans6. Magnetic intensity $H = nI = 500 \text{ x } 3 = 1500\,Am^{-1}$

Magnetic field inside the core $B = \mu H = \mu_0\mu_r H = 4\pi\times10^{-7}\times5000\times1500 = 9.4T$

Ans7. Magnetic field due to a long straight wire is

$$B = \frac{\mu_0 I}{2\pi r} = \frac{4\pi\times10^{-7}\times35}{2\pi\times0.2} = 3.5\times10^{-5}\,T$$

Ans8. Magnetic field et the centre of a circular coil is

$$B = \frac{\mu_0 N I}{2r} = \frac{4\pi\times10^{-7}\times100\times0.4}{2\times8\times10^{-2}} = 3.14\times10^{-4}\,T$$

Ans9. Turns per unit length of the solenoid $n = \frac{N}{l} = \frac{5\times400}{0.8} = 2500$

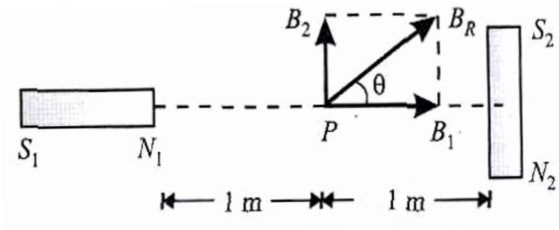
Hence its magnetic field $B = \mu_0 nI = 4\pi\times10^{-7}\times2500\times8 = 2.5\times10^{-2}\,T$

Ans10. The series resistance to be connected with galvanometer to convert it into a voltmeter is

$$R = \frac{V}{I_g} - R_G = \frac{15}{5\times10^{-3}} - 50 = 2950\Omega$$

Level-II

Ans1. The situation is shown in the figure:



The magnetic fields of the two magnets at the midpoint P are

$$B_1 = \frac{\mu_0}{4\pi}\frac{2m}{r^3} = \frac{10^{-7}\times2\times1}{1^3} = 2\times10^{-7}\,T \;(in\,horizontal\,direction)$$

$$B_2 = \frac{\mu_0}{4\pi}\frac{m}{r^3} = \frac{10^{-7}\times1}{1^3} = 10^{-7}\,T \;(in\,vertical\,direction)$$

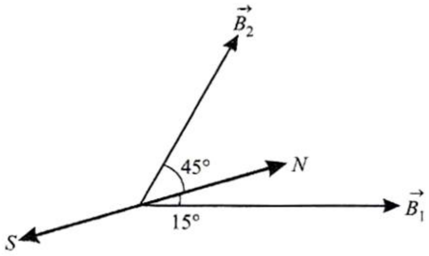
$$B_R = \sqrt{B_1^2 + B_2^2} = \sqrt{5}\times10^{-7}\,T$$

If the resultant field B_R makes angle θ with B_1, then

$$\tan\theta = \frac{B_2}{B_1} = \frac{10^{-7}}{2\times10^{-7}} = 0.5$$

$$\theta = 26.57^0$$

Ans2.



Here, $B_1 = 1.2\times10^{-2}T,\ \theta_1 = 15^0,\ \theta_2 = 60^0 - 15^0 = 45^0$

In equilibrium $\tau_1 = \tau_2$

$$mB_1\,Sin\theta_1 = mB_2\,Sin\theta_2$$

$$B_2 = \frac{B_1\,Sin\theta_1}{Sin\theta_2} = \frac{1.2\times10^{-2}\sin 15^0}{\sin 45^0}$$

Or, $$B_2 = \frac{1.2\times10^{-2}\times0.2588}{0.7071} = 4.4\times10^{-3}\,T$$

Ans3. As the circular portion is three-fourth of a circular loop. Therefore

$$B_O = \frac{3}{4}\frac{\mu_0 I}{2R} = \frac{3\mu_0 I}{8R}$$

Ans4. Current sensitivity $I_s = \dfrac{\alpha}{I}$

Voltage sensitivity $V_s = \dfrac{\alpha}{V} = \dfrac{\alpha}{I\,R_G} = \dfrac{I_s}{R_G}$

New current sensitivity $I_s' = I_s + \dfrac{50}{100}I_s = \dfrac{3}{2}I_s$

So, new voltage sensitivity $V_s' = \dfrac{I_s'}{2R_G} = \dfrac{3I_s/2}{2R_G} = \dfrac{3}{4}V_s$

Thus voltage sensitivity becomes 75% of initial value or decreases by 25%.

Ans5. Resistance of parallel combination of voltmeter and the resistor

$$R_1 = \frac{400\times100}{400+100} = 80\Omega$$

Total resistance of the circuit $R = R_1 + 200 = 280\Omega$

Current in the circuit $I = \dfrac{E}{R} = \dfrac{84}{280} = \dfrac{3}{10}A$

Reading of the voltmeter $V = IR_1 = \dfrac{3}{10}\times80 = 24V$

Total resistance before the voltmeter is connected = 100 + 200 = 300 Ω

Current $I = \dfrac{84}{300} = \dfrac{7}{25}A$

Potential difference across 100 Ω resistor,

$$V = IR = \frac{7}{25} \times 100 = 28V$$

Ans6. For a declination α, torque is

$$\tau = mB \sin \alpha$$

Therefore, $\sin \alpha = \frac{\tau}{mB} = \frac{1.2 \times 10^{-3}}{60 \times 40 \times 10^{-6}} = \frac{1}{2}$

Or $\alpha = 30^0$

Ans7. The magnetic field at the centre of semicircular arc is

$$B = \frac{\mu_0 I}{4r}$$

$$= \frac{4 \times 10^{-7} \times 12}{4 \times 0.02} = 1.9 \times 10^{-4} T$$

According to right hand rule, the field directs normally into the plane of paper.
If the arc is bent in the opposite way, the magnitude of the field remains the same but direction will be opposite (normally out of the plane of the paper)

Ans8. For mid-air suspension, weight of the wire must be balanced by the magnetic force. Hence

$$IlB \sin 90^0 = mg$$

$$B = \frac{mg}{I\,l} = \frac{0.2 \times 9.8}{1.5 \times 2} = 0.65T$$

Ans9. The torque on a magnetic dipole $\tau = M\,B \sin \theta$

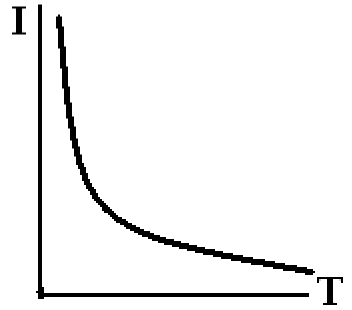
In the equilibrium of the system, torques on both the magnets balance each other. Thus

$$M\,B \sin \theta = \sqrt{3}\,M\,B \sin(90^o - \theta)$$

$$\tan \theta = \sqrt{3} \qquad \Rightarrow \qquad \theta = 60^o$$

Ans10. (i) The slope of I-H curve give susceptibility of the magnetic material. As the slope of *A* is positive and larger, it is ferromagnetic. The slope of *B* is positive and smaller, it is paramagnetic.

(ii) The *I-T* graph for paramagnetic material is shown below:



Level-III

Ans1. Let l be the length of the wire.

When it is bent in form of one turn circular coil,

$$l = 2\pi r_1 \qquad or \qquad r_1 = \frac{l}{2\pi}$$

Therefore $B_1 = \frac{\mu_0 N I}{2R} = \frac{\mu_0 \pi I}{l}$

When it is bent in two turn coil,

$$l = 2\times 2\pi r_1 \qquad or \qquad r_1 = \frac{l}{4\pi}$$

Therefore $B_2 = \frac{\mu_0 N I}{2R} = \frac{4\mu_0 \pi I}{l}$

Hence $B_1 : B_2$ = 4 : 1

Ans2. Current required for full scale deflection

I_g = current for one division deflection x total number of divisions

$$I_g = \frac{1}{10}\times 50 = 5mA$$

If I be the maximum current that a galvanometer can read, then

$$I = \frac{(R_G + S)I_g}{S} \qquad \left[As\ \ S = \frac{I_g R_G}{I - I_g}\right]$$

$$I = \frac{(60+2.5)5}{2.5} = 125mA$$

Ans3. Current through voltmeter $\quad I = \frac{V}{R} = \frac{4}{10\times 10^3} = 4\times 10^{-4}\ A$

Also $\quad I = \frac{Total\,emf}{Total\,resis\tan ce}$

So, $\quad 4\times 10^{-4} = \frac{120}{X + 10^4}$

or $\quad X = 29\times 10^4\Omega$

As the current is very small, the ammeter reading will be too small to be measured accurately. Hence this is an unusual use of voltmeter.

Ans4. $\quad B_{centre} = \frac{\mu_0 N I}{2r} \quad$ and $\quad B_{axial} = \frac{\mu_0 N I r^2}{2(r^2 + x^2)^{3/2}}$

$$\therefore \quad \frac{B_{axial}}{B_{centre}} = \frac{r^3}{(r^2 + x^2)^{3/2}} = \frac{r^3}{r^3}\left[1 + \frac{x^2}{r^2}\right]^{-3/2} = \left[1 - \frac{3}{2}\frac{x^2}{r^2}\right]$$

The fractional decrease in the field

$$\frac{B_{centre} - B_{axial}}{B_{centre}} = \frac{3}{2}\frac{x^2}{r^2}$$

Ans5. For voltmeter of range V,

$$R_1 = \frac{V}{I_g} - R_g \quad or \quad \frac{V}{I_g} = R_1 + R_g$$

For voltmeter of range V/2

$$R_2 = \frac{V}{2I_g} - R_g \qquad or \qquad \frac{V}{2I_g} = R_2 + R_g$$

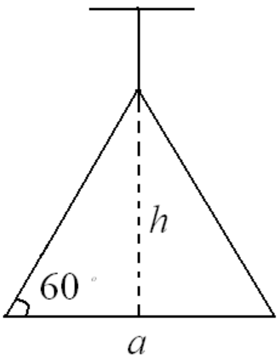
Dividing, we get

$$2 = \frac{R_1 + R_g}{R_2 + R_g} \quad \Rightarrow \quad R_g = R_1 - 2R_2$$

For a voltmeter of range 2V, the series resistance is

$$R = \frac{2V}{I_g} - R_g = 2(R_1 + R_2) - R_g = 3R_1 - 2R_2$$

Ans6. As shown in figure



Area of the triangle $A = \frac{1}{2} \times base \times height = \frac{1}{2} \times a \times a \sin 60^o = \sqrt{3} \times 10^{-4}\ m^2$

Magnetic dipole moment of the coil $M = I\,A = 0.1 \times \sqrt{3} \times 10^{-4} = \sqrt{3} \times 10^{-5}\ Am^2$

Thus torque acting on the coil $\tau = M\,B \sin 90^o = 5\sqrt{3} \times 10^{-7}\ Nm$

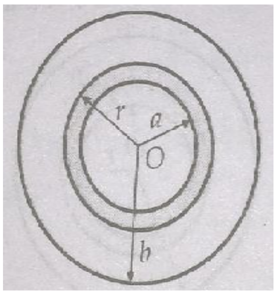
Ans7. Force on the wire $F = I\,B\,d \sin 90^o = I\,B\,d$

This force produces a constant acceleration in the wire. Thus velocity of the wire

$$v = u + a\,t = o + \frac{I\,B\,d}{m} t = \frac{I\,B\,d\,t}{m}$$

Ans8. Let us consider a cylindrical shell of radius r. Current enclosed by this shell

$$I' = \frac{I}{\pi(b^2 - a^2)} \times \pi(r^2 - a^2) = \frac{I(r^2 - a^2)}{(b^2 - a^2)}$$



Using Ampere's circuital law

$$B\,L = \mu_0\, I'$$

$$B \times 2\pi\, r = \mu_0\, \frac{I\,(r^2 - a^2)}{(b^2 - a^2)}$$

$$B = \frac{\mu_0\, I\,(r^2 - a^2)}{2\pi\, r\,(b^2 - a^2)}$$

Ans9. For tension to be zero,

$$B\,I\,L = M\,g \quad \Rightarrow \quad B = \frac{60 \times 10^{-3} \times 10}{5 \times 0.6} = 0.2T$$

When the direction of the current is reversed, total tension is

$T = B\,I\,L + M\,g = 0.4T$

Ans10. For a charge particle to pass undeflected, $q\,E = q\,v\,B$.

Thus $$v = \frac{E}{B} = \frac{50 \times 10^{3}}{100 \times 10^{-3}} = 5 \times 10^{5}\ ms^{-1}$$

When the beam strikes the screen, its final momentum becomes zero. Thus the force exerted,

$$F = \frac{m\,v}{t} = \frac{m\,v}{q\,/\,I} = \frac{m\,v\,I}{n\,e}$$