

Electrostatics

IMPORTANT FORMULAE IN ELECTROSTATICS

1. Electrostatic force between two charges

$$F = K \cdot \frac{q_1 q_2}{r^2} = \frac{1}{4\pi\epsilon_0\epsilon_r} \cdot \frac{q_1 q_2}{r^2}$$

For air, $\epsilon_r = 1$

$$F_{\text{air}} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_2}{r^2} = 9 \times 10^9 \frac{q_1 q_2}{r^2}$$

2. Electric field intensity due to a point charge, $\vec{E} = \lim_{q_0 \rightarrow 0} \frac{\vec{F}}{q_0}$

3. Electric field intensity due to infinite linear charge density (λ)

$$E = \frac{1}{4\pi\epsilon_0} \cdot \frac{2\lambda}{r}$$

4. Electric field intensity near an infinite thin sheet of surface charge density σ

$$E = \frac{\sigma}{2\epsilon_0}$$

For thick sheet = $\frac{\sigma}{\epsilon_0}$.

5. Electric potential, $V = \lim_{q_0 \rightarrow 0} \frac{W}{q_0}$

Electric potential due to a point charge, $V = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r}$

6. Relation between electric field and potential $E = -\frac{dV}{dr} = \frac{V}{r}$ (numerically)

7. Dipole moment, $\vec{P} = q \cdot 2\vec{l}$

8. Torque on a dipole in uniform electric field, $\vec{\tau} = \vec{p} \times \vec{E}$.

9. Potential energy of dipole, $U = -\vec{p} \cdot \vec{E} = -pE \cos \theta$

10. Work done in rotating the dipole in uniform electric field from orientation Q_1 to Q_2 is

$$W = U_2 - U_1 = pE(\cos \theta_1 - \cos \theta_2)$$

11. Electric field due to a short dipole

(i) at axial point, $E_{\text{axis}} = \frac{1}{4\pi\epsilon_0} \cdot \frac{2p}{r^3}$

(ii) at equatorial point, $E_1 = \frac{1}{4\pi\epsilon_0} \cdot \frac{p}{r^3}$

12. Electric potential due to a short dipole

(i) At axial point, $V_{\text{axis}} = \frac{1}{4\pi\epsilon_0} \cdot \frac{p}{r^2}$

(ii) At equatorial point, $V_{\blacksquare} = 0$.

13. Dielectric constant, $K = \frac{\epsilon}{\epsilon_0} = \frac{C_{\text{med}}}{C_{\text{air}}}$

14. Capacitance of parallel plate capacitor

(i) $C = \frac{A\epsilon_0 K}{d}$, in medium of dielectric constant K

(ii) $C = \frac{A\epsilon_0}{d - t(1 - \frac{1}{K})}$; if space between plate partially filled with dielectric of thickness t.

15. Combination of capacitors :-

(i) In series, $\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$, $q_1 = q_2 = q_3$, $V = V_1 + V_2 + V_3$

(ii) In parallel, $C = C_1 + C_2 + C_3$, $q = q_1 + q_2 + q_3$, $V_1 = V_2 = V_3 = V$

16. Energy stored by capacitor

$$U = \frac{1}{2} CV^2 = \frac{Q^2}{2C} = \frac{1}{2} QV$$

17. Electrostatic energy density

$$\vartheta_e = \frac{1}{2} \epsilon_0 E^2, \text{ in air}$$

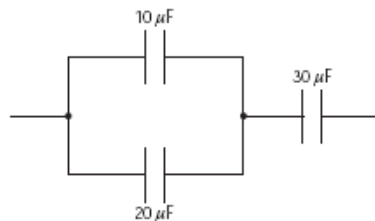
$$\vartheta_e = \frac{1}{2} \epsilon E^2, \text{ in medium}$$

18. Total electric flux, $\Phi = \oint \vec{E} \cdot \vec{ds} = \frac{1}{\epsilon_0} \times \text{net charge enclosed by the surface}$

NUMERICALS

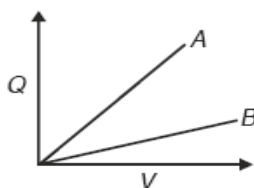
LEVEL I

1. What is the charge acquired by a body when 1 million electrons are transferred to it?
2. An attractive force of 5N is acting between two charges of $+2.0 \mu\text{C}$ & $-2.0 \mu\text{C}$ placed at some distance. If the charges are mutually touched and placed again at the same distance, what will be the new force between them?
3. A charge of $+3.0 \times 10^{-6} \text{ C}$ is 0.25 m away from a charge of $-6.0 \times 10^{-6} \text{ C}$.
 - a. What is the force on the $3.0 \times 10^{-6} \text{ C}$ charge?
 - b. What is the force on the $-6.0 \times 10^{-6} \text{ C}$ charge?
4. An electric dipole consist of a positive and a negative charge of $4\mu\text{C}$ each placed at a distance of 5mm. Calculate dipole moment.
5. Three capacitors of capacitances $2\mu\text{F}$, $3\mu\text{F}$ and $4\mu\text{F}$ are connected in parallel. What is the equivalent capacitance of the combination? Determine charge on each capacitor, if the combination is connected to 100V supply?
6. An electric dipole with dipole moment $4 \times 10^{-9} \text{ C}\cdot\text{m}$ is aligned at 30° with direction of electric field of magnitude $5 \times 10^4 \text{ N/C}$. Calculate the magnitude of the torque acting on the dipole.
7. A point charge of $2\mu\text{C}$ is at the centre of cubic Gaussian surface 9.0 cm in edge. What is the net electric flux through the surface?
8. What is the amount of work done in moving a 200nC charge between two points 5 cm apart on an equipotential surface?
9. How much work must be done to charge a $24 \mu\text{F}$ capacitor, when the potential difference between the plates is 500 V?
10. What is the equivalent capacity of the network given below?

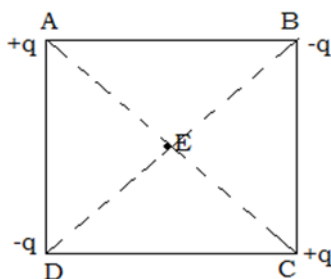


LEVEL II

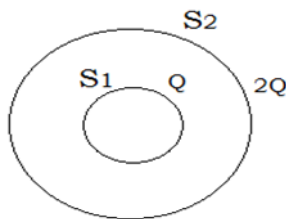
1. What is the work done in moving a charge of $100\mu\text{C}$ through a distance of 1cm along the equatorial line of dipole?
2. The given graph shows that variation of charge q versus potential difference V for two capacitors C_1 and C_2 . The two capacitors have same plate separation but the plate area of C_2 is double than that of C_1 . Which of the lines in the graph correspond to C_1 and C_2 and why?



3. Two point charges $5\mu\text{C}$ and $-4\mu\text{C}$ are separated by a distance of 1 m in air. At what point on the line joining the charges is the electric potential zero?
4. Two charges $+5\mu\text{C}$ and $+20\mu\text{C}$ are placed 15 cm apart. At what point on the line joining the two charges is the electric field zero?
5. Two charges $+16\mu\text{C}$ and $-9\mu\text{C}$ are placed 8 cm apart. At what point on the line joining the two charges is the electric field zero?
6. A 600 pF capacitor is charged by a 200 V supply. It is then disconnected from the supply and is connected to another uncharged 600 pF capacitor. How much electrostatic energy is lost in the process.
7. Keeping the voltage of the charging source constant, what will be the percentage change in the energy stored in a parallel plate capacitor if the separation between its plates were to be decreased by 10%.
8. Four charges are placed at the vertices of a square of side d as shown in the figure. (i) Find the work done to put together this arrangement. (ii) A charge q_0 is brought to the center E of the square, the four charges being held fixed at its corners. How much extra work is needed to do this?



9. If S_1 and S_2 are two hollow spheres enclosing charges Q and $2Q$ respectively as shown in the figure

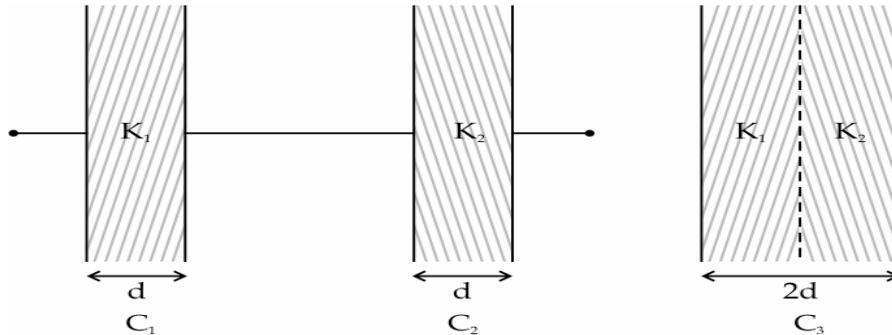


- (i) What is the ratio of the electric flux through S_1 and S_2 ?
 - (ii) How will the flux through the sphere S_1 change, if a medium of dielectric constant 5 is filled in the space inside S_1 .
10. A charge of $24\mu\text{C}$ is given to a hollow sphere of radius 0.2m. Find the potential
 - (i) at the surface of the sphere, and
 - (ii) at a distance of 0.1 m from the centre of the sphere.
 - (iii) at the centre

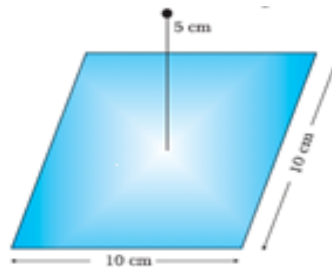
LEVEL III

1. A slab of material of dielectric constant κ has the same area as the plates of a parallel plate capacitor but has a thickness $3d/4$, where d is the separation of the plates. How is the capacitance changed when the slab is inserted between the plates?
2. A parallel plate capacitor with air between the plates has a capacitance of $8\mu\text{F}$. What will be the capacitance if the distance between the plates is doubled and the space between them is filled with a substance of dielectric constant $K=6$?

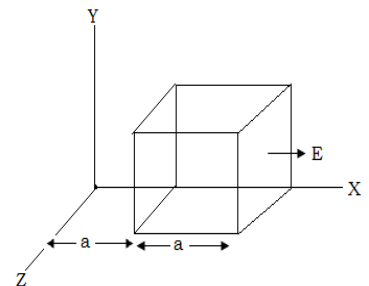
- Two dipoles, made from charges $\pm q$ and $\pm Q$, respectively, have equal dipole moments. Give the (i) ratio between the 'separations' of these two pairs of charges (ii) angle between the dipole axis of these two dipoles.
- The capacitors C_1 , and C_2 , having plates of area A each, are connected in series, as shown. Compare the capacitance of this combination with the capacitor C_3 , again having plates of area A each, but 'made up' as shown in the figure.



- A point charge $+10\mu\text{C}$ is at a distance 5cm directly above the centre of a square of side 10cm as shown in fig. What is the magnitude of flux through the square?



- Calculate equivalent capacitance of the given network and determine the charge and voltage across each capacitor.
- Two identical charges, Q each are kept at a distance r from each other. A third charge q is placed on the line joining the two charges such that all the three charges are in equilibrium. What is magnitude, sign and position of the charge q ?
- ABCD is a square of side 5m . Charges of $+50\text{C}$, -50C and $+50\text{C}$ are placed at A, C and D respectively. Find the magnitude of resultant electric field at B.
- A cube with each side a is kept in electric field given by $E = Cx$ as shown in the figure where C is a positive dimensional constant. Find
 - The electric flux through the cube, and
 - The net charge inside the cube.



- Two parallel plate capacitor X and Y have same area of plates and same separation between them. X has air between the plates whereas Y has a dielectric of constant $k=4$
 - Calculate capacitance of each capacitor if equivalent capacitance is $4\mu\text{F}$.
 - Calculate potential difference between the plates of X and Y.
 - What is the ratio of electrostatic energy stored in X and Y.

UNIT: I ELECTROSTATICS

ANSWERS

LEVEL I

1. $Q = Ne \ 1.6 \times 10^{-13}C$
2. $F=0$
3. $F_{AB} = F_{BA}=2.736N$
4. $P=2 \times 10^{-8} \text{ C-m}$
- 5.
6. $10^{-4}Nm$
7. $2,26 \times 10^5 Nm^2/C$
8. $W=0$
9. $W=3J$
10. $C=15\mu F$

LEVEL II

1. 0
2. A
3. $\frac{5}{9}m$ from $5\mu C$ charge
4. 5 cm from $5 \mu C$ charge
5. 24cm from $-9 \mu C$ charge
6. $6 \times 10^{-6} \text{ J}$
7. 11.11%
8. $\frac{q^2}{4\pi\epsilon_0} (4 - \sqrt{2})$, 0
9. 1:3, $\phi = \frac{Q}{5\epsilon_0}$
10. (i) 1.08×10^6V (ii) 1.08×10^6V (iii) 1.08×10^6V

LEVEL III

1. $\frac{4k}{k+3} C_0$
2. $24 \mu F$
3. $q \ a=Q \ A$ or $a/A=Q/q \ \theta = 0$
4. $C_3= C_{eq}$
5. $1.88 \times 10^5 Nm^2/C$
6. $\frac{200}{3} pF, 100 \text{ V}, 50V, 50V, 200V, 10^{-8}C, 10^{-8}C, 10^{-8}C, 2 \times 10^{-8} \text{ C}$
7. $Q/4$, Positive, $r/2$
8. $2.7 \times 10^{10} N/C$
9. $a^3C \text{ N-m}^2/C, a^3C\epsilon_0 \text{ Coulombs.}$
10. $C_x=5\mu F \ C_y= 20Mf$