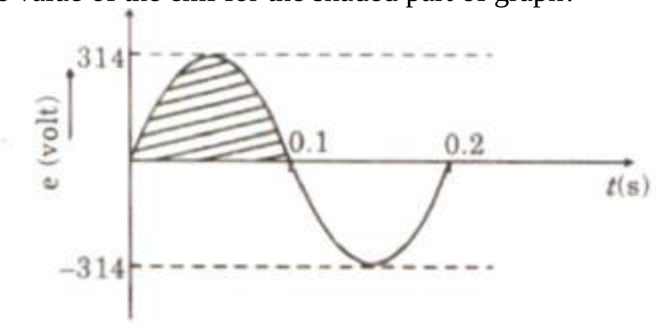
EMI and AC
All possible formulae at a glance

Physical Quantity	Formula	SI unit	Dimension
Magnetic flux (ϕ)	$\vec{B}\bullet\vec{A} = BA\cos\theta = \int \vec{B}\bullet d\vec{A}$	Wb = Tm^2	$[ML^2T^{-2}A^{-1}]$
Induced emf (e)	$\varepsilon = -\frac{d\phi}{dt}$ Induced current $i = \frac{\varepsilon}{R} = -\frac{N}{R}\frac{d\phi}{dt}$ Induced charge $q = i\Delta t = -\frac{N}{R}\Delta\phi$ Motional emf induced in a straight conductor (i) Linear motion = Blv (ii) Rotation about one end = $Bl^2\omega/2$	Volt	$[ML^2T^{-1}A^{-1}]$
Self-inductance	$L = \phi/I$ and $L = \frac{\lvert\varepsilon\rvert}{dI/dt}$ Self-inductance of a long solenoid $L = \mu_r \mu_0 n^2 A\, l$	Henry	$[ML^2T^{-2}A^{-2}]$
Mutual inductance	$M_{12} = \phi_2/I_1$ and $M_{12} = \frac{\lvert\varepsilon_2\rvert}{dI_1/dt}$ Mutual-inductance of two long co-axial solenoids $M_{12} = \mu_0 n_1 n_2 \pi r^2 l$, $M_{12} = \sqrt{^3}L_1L_2{}'$	Henry	$[ML^2T^{-2}A^{-2}]$
Magnetostatic energy stored	U = ½ LI^2	Joule	$[ML^2T^{-2}]$
Alternating current and voltage	$e = E_0 \sin(\omega t+\phi)$ or $e = E_0\cos(\omega t+\phi)$ $i = I_0 \sin(\omega t+\phi)$ or $i = I_0 \cos(\omega t+\phi)$ $I_{rms} = I_o/\sqrt{2} = 0.707\, I_0$ and $E_{rms} = E_o/\sqrt{2} = 0.707\, E_0$		
Phase relationship	**For R** : No phase difference bet^n V and I **For L**: Voltage leads the current by $\pi/2$ **For C**: Current leads the voltage by $\pi/2$ **For LCR circuit**: **if f > f_r** $\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$ or $\phi = \tan^{-1}\left(\frac{V_L - V_C}{V_R}\right)$ **If f<f_r** $\phi = \tan^{-1}\left(\frac{X_C - X_L}{R}\right)$ or $\phi = \tan^{-1}\left(\frac{V_C - V_L}{V_R}\right)$	Unit less	Dimensionless
Reactance and impedance	Inductive reactance $X_L = \omega L$ Capacitive reactance $X_C = 1/\omega C$ Impedance of LR circuit $Z = \sqrt{\{X_L^2 + R^2\}}$ Impedance of RC circuit $Z = \sqrt{\{X_C^2 + R^2\}}$ Impedance of LCR circuit $Z = \sqrt{\{(X_L - X_C)^2 + R^2\}}$	Ohm	$[ML^2T^{-1}A^{-2}]$
Resonance frequency	$f_r = \frac{1}{2\pi\sqrt{LC}}$, angular frequency $\omega_r = \frac{1}{\sqrt{LC}}$	Hertz, rad/s	$[T^{-1}]$
Quality factor	Q = $\frac{1}{R}\sqrt{\frac{L}{C}} = \frac{\omega_r}{2\Delta\omega} = \frac{\omega_r L}{R} = \frac{1}{\omega_r CR}$	Unit less	Dimensionless
Power dissipated in ac	In pure inductor and capacitor: Zero	Watt	$[ML^2T^{-3}]$

circuit	In pure resistive circuit: $I^2R/2$ In a combination of L,C and R: $V_{rms}I_{rms}\cos\phi$		
Power factor	$\cos\phi = R/Z$	Unit less	Dimensionless
Wattles current	$I_{rms}\cos\phi$	Ampere	[A]
Frequency of LC oscillations	$f_r = \dfrac{1}{2\pi\sqrt{LC}}$	Hertz	$[T^{-1}]$
Energy of ideal LC oscillator	$\frac{1}{2} Q^2/C + \frac{1}{2} LI^2 = \frac{1}{2} Q_0^2/C$	Joule	$[ML^2T^{-2}]$
Transformation ratio and efficiency of transformer	$\dfrac{v_S}{v_P} = \dfrac{N_S}{N_P}$ Efficiency:- $\eta = \dfrac{v_s I_s}{v_p I_p} = \dfrac{P_0}{P_i}$	Unit less	Dimensionless

LEVEL I

1. What is the self-inductance of a coil in which magnetic flux of 40mWb is produced when 2A current flow through it?
2. If the self inductance of an air core inductor increases from 0.01mH. to 100mH on introducing an iron core into it. What is relative permeability of the core used?
3. What is the power dissipated in an a.c circuit in which voltage and current are given by $V=230\sin(\omega t + \pi/2)$ and $I=10\sin\omega t$?
4. When a lamp is connected to an a.c. supply it light with the same brightness as when connected to a 12V d.c battery. What is the peak value of alternating voltage?
5. Find the capacitance of the capacitor that would have reactance of 100 □□when used with a.c. source of frequency $\frac{5}{\pi}$ kHz ?
6. What is the average value of the emf for the shaded part of graph?



7. In a series LCR circuit the voltage across an inductor, a capacitor and a resistor are 20V, 20V and 60V respectively. What is the phase difference between the applied voltage and the current in the circuit?
8. A circular coil of radius 8 cm and 20 turns rotates about its vertical diameter with an angular speed of 50/s in a uniform horizontal magnetic field of magnitude 3×10^{-2} T. Find the max. and average value of the emf induced in the coil.
9. The instantaneous current from an a.c. source is $I = 5\sin(314t)$ ampere. What are the average and rms values of the current?
10. An inductor L, a capacitor 20 μF, a resistance 10□.are connected in series with an ac source of frequency 50 Hz. If the current is in phase with voltage, calculate the Inductance L.

LEVEL II

Q1. A conductor of length 1.0 m falls freely under gravity from a height of 10 m so that it cuts the lines of force of the horizontal component of earth's magnetic field of $3x10^{-5}$ Wbm^{-2}. Find the emf induced in the conductor.

Q2. A 0.4 m long straight conductor is moved in a magnetic field of induction 0.9 Wbm^{-2} with velocity of 7 ms^{-1}. Calculate the maximum emf induced in the conductor.

Q3. A metal disc of radius 200 cm is rotated at a constant angular speed of 60 $rads^{-1}$ in a plane at right angles to an external field of magnetic induction 0.05 Wbm^{-2} .Find the emf induced between the centre and a point on the rim.

Q4. Find the maximum value of current when an inductance of one Henry is connected to an a.c. source of 200 volts, 50 Hz.

Q5. What is the inductive reactance of a coil if current through it is 800 mA and the voltage across it is 40 V?

Q6. A transformer has 300 primary turns and 2400 secondary turns .If the primary supply voltage is 230 V, what is the secondary voltage?

Q7. A transformer of 100% efficiency has 500 turns in the primary and 10,000 turns in the secondary coil. If the primary is connected to 220 V supply, what is the voltage across the secondary coil?

Q8. A capacitor in series with a resistance of 30 ohm is connected to a.c. mains. The reactance of the capacitor is 40 ohm. Calculate the phase difference between the current and the supply voltage.

Q9. Determine the impedance of a series LCR-circuit if the reactance of C and L are 250 ohm and 220 ohm respectively and R is 40 ohm.

Q10. A series circuit with L=0.12 H, C=0.48 mF and R=25 ohm is connected to a 220 V variable frequency power supply. At what frequency is the circuit current maximum?

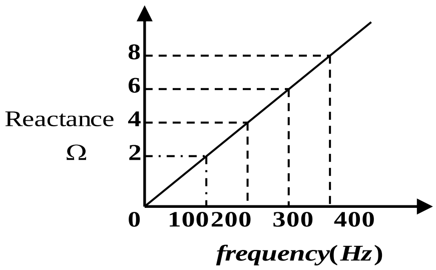
LEVEL III

1. A bulb of resistance 10Ω, connected to an inductor of inductance L, is in series with an ac source marked 100V, 50Hz. If the phase angle between the voltage and current is $\frac{\pi}{4}$ radian, calculate the value of L.

2. Figure shows how the reactance of an inductor varies with frequency.

   (a) Calculate the value of inductance of the inductor using the information given in the graph.

   (b) If this inductor is connected in series to a resistor of 8 ohm, find what would be the impedance at 300 Hz.



3. In a series RC circuit, R = 30 Ω, C = 0.25 μ F, V = 100 V and ω = 10,000 radian per second. Find the current in the circuit and calculate the voltage across the resistor and the capacitor. Is the algebraic sum of these voltages more than the source voltage? If yes, resolve the paradox.

4. When an alternating voltage of 220V is applied across a device *X*, a current of 0.5A flows through the circuit and in phase with the applied voltage. When the same voltage is applied across another device *Y*, the same current again flows through the circuit but it leads the applied voltage by $\pi/2$ radians. (i) Name the devices *X* and *Y*, (ii) Calculate the current flowing in the circuit when the same voltage is applied across the series combination of *X* and *Y*.

5. In the series LCR circuit, suppose R = 300 Ω, L = 60 mH, C = 0.5 μ F. An ac source of emf 50 V, angular frequency 10,000 rad/s is connected across the combination. Find the reactance X_L, and X_C, the impedance Z, the current amplitude I, the phase angle φ, and the voltage amplitude across each circuit element.

6. In a series RC circuit with an AC source, R = 300 Ω, C = 25 μF, e_0 = 50 V and $\nu = \frac{50}{\pi}$ Hz, find the peak current and the average power dissipated in the circuit.

7. An inductor 200 mH, capacitor 500 μ F, resistor 10 Ω are connected in series with a 100 V, variable frequency ac source. Calculate the (i) frequency at which the power factor of the circuit is unity; (ii) current amplitude at this frequency; (iii) Q-factor.

8. A resistor of resistance 400 Ω, and a capacitor of reactance 200 Ω, are connected in series to a 220 V, 50 Hz ac source. If the current in the circuit is 0.49 A, find the (i) voltage across the resistor and capacitor (ii) value of inductance required so that voltage and current are in phase.

9. A resistor of 200 Ω and a capacitor of 15 μ F are connected in series to a 220 V, 50 Hz ac source. (a) Calculate the current in the circuit; (b) calculate the voltage (rms) across the resistor and the capacitor. Is the algebraic sum of these voltages more than the source voltage? If yes, resolve the paradox.

10. A town is situated 15 km away from a power plant generating power at 440V, requires 800 kW of electric power at 220V. The resistance of the two wire line carrying power is 0.5 ohm per km. The town gets power from the line through a 4000-220V step down transformer at a substation in the town.

(i) Find the line power losses in the form of heat.

(ii) How much power must the plant supply, assuming there is negligible power loss due to leakage?

(iii) Characterize the step up transformer at the plant.

11. An ac generator consists of a coil of 50 turns and area 2.5 m^2 rotating at an angular speed of 60 rad/s in a uniform magnetic field B = 0.3 T between two fixed pole pieces. The resistance of the circuit including that of the coil is 500 Ω. Determine the Calculate

(i) maximum current drawn from the generator. (ii) maximum power dissipation in the coil.

(iii) What will be orientation of the coil with respect to the magnetic field to have (*a*) maximum, (*b*) zero magnetic flux? (iv) Would the generator work if the coil was stationary and instead the pole pieces rotated together with the same speed as above?

12. A circular coil having 20 turns, each of radius 8 cm, is rotating about its vertical diameter with an angular speed of 50 radian/s in a uniform horizontal magnetic field of magnitude 30 mT. Obtain the maximum average and rms value of the emf induced in the coil. If the coil forms a closed loop of resistance 10 Ω, how much power is dissipated as heat in it?

13. An athlete peddles a stationary tricycle whose pedals are attached to a coil having 100 turns each of area 0.1m^2. The coil, lying in the X-Y plane, is rotated, in this plane, at the rate of 50 rpm, about the Y-axis, in a region where a uniform magnetic field, $\vec{B}$ = (0.01) $\hat{k}$ tesla, is present. Find the (i) maximum emf (ii) average e.m.f generated in the coil over one complete revolution.

Answers of Level I

1. 20 mH. 2. *1000* 3. zero 4. *16.92V* 5. $10^{-6} F$ 6. *200V* 7. *0 rad.* 8. ε_{max} = *0.6 V* ; ε_{av} *=0 V* 9. average value of current = 0, rms value of current= $\frac{5}{\sqrt{2}}$ A 10. 2X 10^5 H]

Answers of Level II

Q1:- $4.2\text{x}10^{-4}$ V , Q2:- 2.52 V , Q3:- 6 V , Q4:- 0.9 A , Q5:- 50 ohm , Q6:- 1.84 kV , Q7:- 4400 V , Q8:- $\tan^{-1} 4/3$, Q9:- 50 ohm , Q10:- 21 Hz

Answers of Level III

Ans1. Cos $\phi = \frac{R}{Z}$ $\Rightarrow \cos \frac{\pi}{4} = \frac{R}{\sqrt{R^2 + X_L^2}}$ $\Rightarrow X_L = R$ $\Rightarrow L = 3.14 \text{ x } 10^{-2}$ H

Ans2. (a) $L = \frac{X_L}{2\pi\nu}$ (b) at 300 Hz; $X_L = 6\ \Omega$, $R = 8\ \Omega$ $\Rightarrow Z = \sqrt{R^2 + X_L^2} = 10\ \Omega$

Ans3. $X_C = \frac{1}{2\pi\nu C}$ =400 Ω; $Z = \sqrt{R^2 + X_C^2} = 500\ \Omega$; $i = \frac{e}{Z} = \frac{100}{500}$ =0.2 A

rms voltage drop on R = $V_R = i_{rms} R$ = 60 V; rms voltage drop on C = $V_C = i_{rms} X_C$ = 80 V

$V_L + V_C$ = 140 V > e; This paradox occurs because being phasor quantities, V_R and V_C cannot be added algebraically.

Ans4. Since current and voltage are in phase, so, device X is resistor. $R = \frac{220}{0.5} = 440\Omega$

For device Y, current leads the voltage in phase by $\frac{\pi}{2}$, so, Y is a capacitor. $X_C = \frac{220}{0.5} = 440\Omega$

When R and C are connected in series then $i_{rms} = \frac{e_{rms}}{\sqrt{R^2 + X_C^2}}$ =0.3535 A

Ans5. $X_L = \omega L = 600\ \Omega$; $X_C = \frac{1}{\omega C}$ =200 Ω; $Z = \sqrt{R^2 + (X_L - X_C)^2} = 500\ \Omega$; $i_0 = i_{rms}\sqrt{2}$ = 0.1414 A

Phase angle, $\phi = \cos^{-1} \frac{R}{Z} = \cos^{-1} (0.6)$; V_o (across R) = i_0 R = 42.42 V; V_o (across L) = $i_0 X_L$ = 84.84 V;

V_o (across C) = $i_0 X_C$ = 28.28 V

Ans6. Peak current = $i_0 = \frac{e_0}{Z} = \frac{e_0}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}} = 0.1A$;

Power dissipated = $e_{rms}\, i_{rms} \cos\phi = \frac{e_0}{\sqrt{2}} \frac{i_0}{\sqrt{2}} \frac{R}{Z}$ = 1.5 Watt

Ans7. Power factor is unity when circuit is in resonance for which the frequency is $\nu = \frac{1}{2\pi\sqrt{LC}}$ =15.9 Hz

At resonant frequency, Z = R; Current amplitude = $i_0 = \frac{e_0}{Z} = \frac{e_0}{R} = \frac{e_{rms}\sqrt{2}}{R}$ =14.14 A

Quality factor Q = $\frac{1}{R}\sqrt{\frac{L}{C}} = \frac{\omega_r}{2\Delta\omega}$ =2

Ans8. (i) $V_R = i_{rms} R$ = 196 V; $V_C = i_{rms} X_C$ = 98 V (ii) If the circuit is LCR as V and I are in phase then LCR circuit is in resonance for which $X_L = X_C$ $\Rightarrow$ L = 0.628 H

Ans 9. $X_C = \frac{1}{2\pi\nu C}$ =210 Ω; $i_{rms} = \frac{e_{rms}}{Z} = \frac{e_{rms}}{\sqrt{R^2 + X_C^2}}$ =0.76 A; rms potential drop on R = $V_R = i_{rms} R$ = 152 V

rms potential drop on C = $V_C = i_{rms} X_C$ = 159.6 V; Sum of V_L and V_C = 312 V which is > than applied voltage 200 V; As V_R and V_C are phasor quantities, so, they cannot be added algebraically.

Ans 10. (i)Assuming transformer to be ideal, the current in the transmission line I = 800000/4000 = 200 A

Therefore line power loss = $I^2R = (200)^2$x15 = 600 kW (ii) the plant should supply 1400 kW power (iii) the output voltage of step up transformer at the plant = 1400000/200 = 7000 V. therefore the characterization of step up transformer is 440 V – 7000 V

Ans11. (i) Maximum current $i_0 = \frac{e_0}{R} = \frac{NBA\omega}{R}$ =4.5 A; (ii) Maximum power dissipation = $i_0^2 R$ = 10125 W

(iii) (a) For maximum flux, coil must be ⊥ to $\vec{B}$; (b) for minimum flux, coil must be || to $\vec{B}$

(iv) If pole pieces are rotates then also, the flux of coil changes thus generator works.

Ans 12. $e_{ave} = 0$; $e_{rms} = \frac{e_0}{\sqrt{2}} = \frac{NBA\omega}{R}$ =0.4242 V; Power dissipated = $i_{rms}^2 R = \frac{e_{rms}^2}{R} = \frac{e_0^2}{2R}$ =18 x 10^{-3} W

Ans 13. (i) Maximum emf 'e' generated in the coil is e = NBAω = 0.52 V (ii) The average emf generated in the coil over one complete revolution = 0